Domain adaptation with optimal transport

from mapping to learning with joint distribution

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Introduction

Supervised learning

Amazon

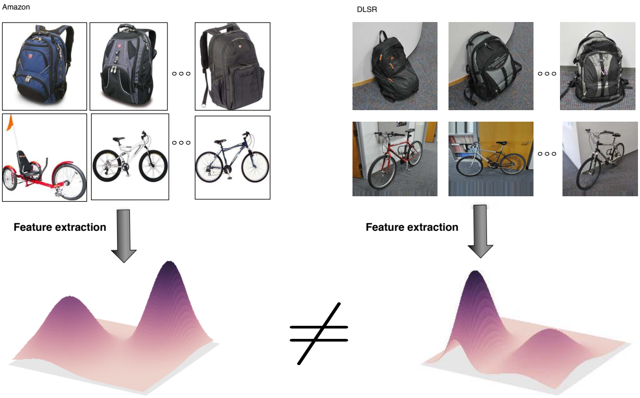
Traditional supervised learning

- We want to learn predictor such that $y \approx f(\mathbf{x})$.
- Actual $\mathcal{P}(X, Y)$ unknown.
- We have access to training dataset $(\mathbf{x}_i, y_i)_{i=1,\ldots,n}$ $(\widehat{\mathcal{P}}(X, Y))$.
- We choose a loss function $\mathcal{L}(y, f(\mathbf{x}))$ that measure the discrepancy.

Empirical risk minimization
We week for a predictor f minimizing

$$\min_f \left\{ \mathop{\mathbb{E}}_{(\mathbf{x},y)\sim\widehat{\mathcal{P}}} \mathcal{L}(y, f(\mathbf{x})) = \sum_j \mathcal{L}(y_j, f(\mathbf{x}_j)) \right\} \tag{1}$$

- Well known generalization results for predicting on new data.
- Loss is usually $\mathcal{L}(y, f(\mathbf{x})) = (y - f(\mathbf{x}))^2$ for least square regression and is $\mathcal{L}(y, f(\mathbf{x})) = \max(0, 1 - yf(\mathbf{x}))^2$ for squared Hinge loss SVM.

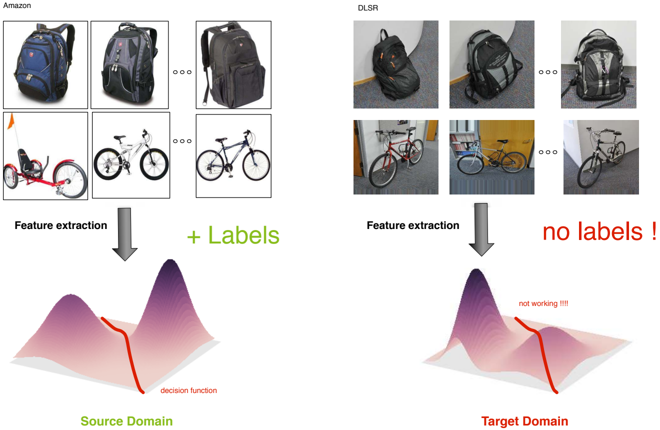


Probability Distribution Functions over the domains

Our context

- Classification problem with data coming from different sources (domains).
- Distributions are different but related.

Unsupervised domain adaptation problem



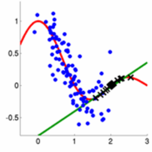
Problems

- Labels only available in the **source domain**, and classification is conducted in the **target domain**.
- Classifier trained on the source domain data performs badly in the target domain

Domain adaptation short state of the art

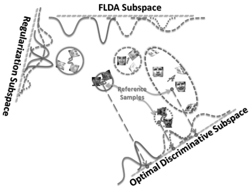
Reweighting schemes [Sugiyama et al., 2008]

- Distribution change between domains.
- Reweigh samples to compensate this change.



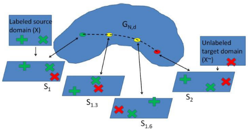
Subspace methods

- Data is invariant in a common latent subspace.
- Minimization of a divergence between the projected domains [Si et al., 2010].
- Use additional label information [Long et al., 2014].

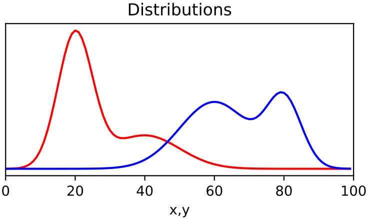


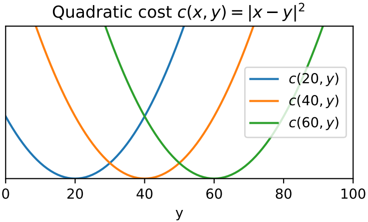
Gradual alignment

- Alignment along the geodesic between source and target subspace [R. Gopalan and Chellappa, 2014].
- Geodesic flow kernel [Gong et al., 2012].



Optimal transport (Monge formulation)



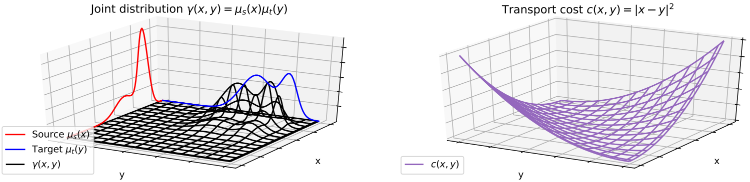


- Probability measures μ_s and μ_t on and a cost function $c : \Omega_s \times \Omega_t \rightarrow \mathbb{R}^+$.
- The Monge formulation [Monge, 1781] aim at finding a mapping $T : \Omega_s \rightarrow \Omega_t$

$$\inf_{T\#\mu_s=\mu_t} \quad \int_{\Omega_s} c(\mathbf{x}, T(\mathbf{x}))\mu_s(\mathbf{x})d\mathbf{x} \tag{2}$$

- Non-convex optimization problem, mapping does not exist in the general case.
- [Brenier, 1991] proved existence and unicity of the Monge map for $c(x, y) = \|x - y\|^2$ and distributions with densities.

Optimal transport (Kantorovich formulation)



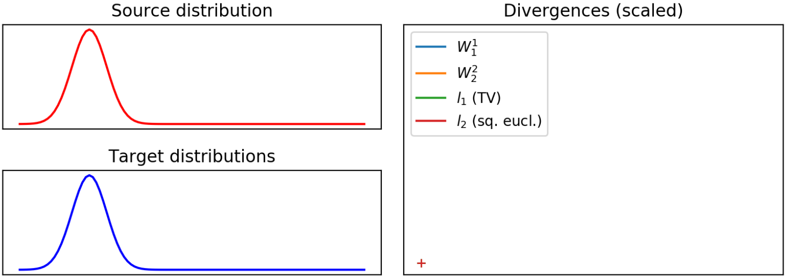
- The Kantorovich formulation [Kantorovich, 1942] seeks for a probabilistic coupling $\gamma \in \mathcal{P}(\Omega_s \times \Omega_t)$ between Ω_s and Ω_t:

$$\boldsymbol{\gamma}_0 = \underset{\boldsymbol{\gamma}}{\operatorname{argmin}} \int_{\Omega_s \times \Omega_t} c(\mathbf{x}, \mathbf{y})\boldsymbol{\gamma}(\mathbf{x}, \mathbf{y}) d\mathbf{x} d\mathbf{y}, \tag{3}$$

$$\text{s.t.} \quad \boldsymbol{\gamma} \in \mathcal{P} = \left\{ \boldsymbol{\gamma} \geq \mathbf{0}, \int_{\boldsymbol{\Omega_t}} \boldsymbol{\gamma}(\mathbf{x}, \mathbf{y}) \mathbf{dy} = \boldsymbol{\mu_s}, \int_{\boldsymbol{\Omega_s}} \boldsymbol{\gamma}(\mathbf{x}, \mathbf{y}) \mathbf{dx} = \boldsymbol{\mu_t} \right\}$$

- γ is a joint probability measure with marginals μ_s and μ_t.
- Linear Program that always have a solution.

Wasserstein distance



Wasserstein distance

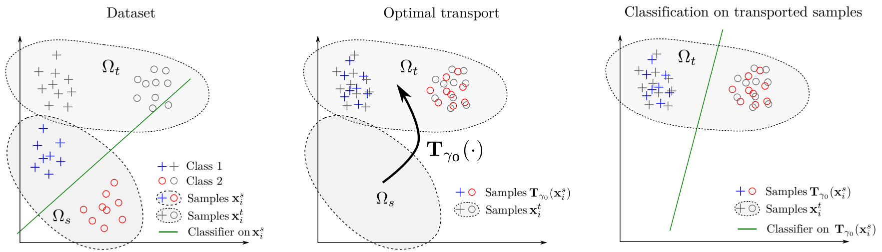
$$W_p^p(\mu_s, \mu_t) = \min_{\gamma \in \mathcal{P}} \int_{\Omega_s \times \Omega_t} c(\mathbf{x}, \mathbf{y}) \gamma(\mathbf{x}, \mathbf{y}) d\mathbf{x} d\mathbf{y} = E_{(\mathbf{x}, \mathbf{y}) \sim \gamma}[c(\mathbf{x}, \mathbf{y})] \tag{4}$$

where $c(\mathbf{x}, \mathbf{y}) = \|\mathbf{x} - \mathbf{y}\|^p$

- A.K.A. Earth Mover's Distance (W_1^1) [Rubner et al., 2000].
- Do not need the distribution to have overlapping support.
- Subgradients can be computed with the dual variables of the LP.
- Works for continuous and discrete distributions (histograms, empirical).

Optimal transport for domain adaptation

Optimal transport for domain adaptation



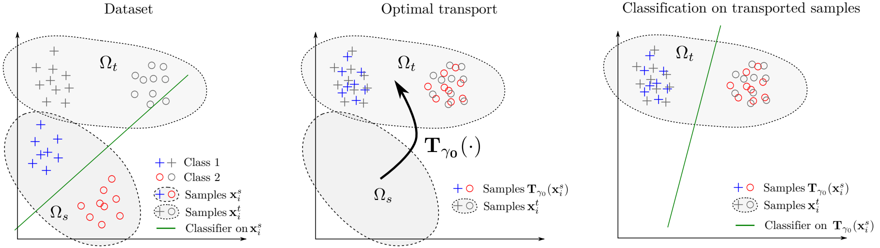
Assumptions

- There exist a transport in the feature space $\mathbf{T}$ between the two domains.
- The transport preserves the conditional distributions:

$$P_s(y|\mathbf{x}_s) = P_t(y|\mathbf{T}(\mathbf{x}_s)).$$

3-step strategy [Courty et al., 2016a]

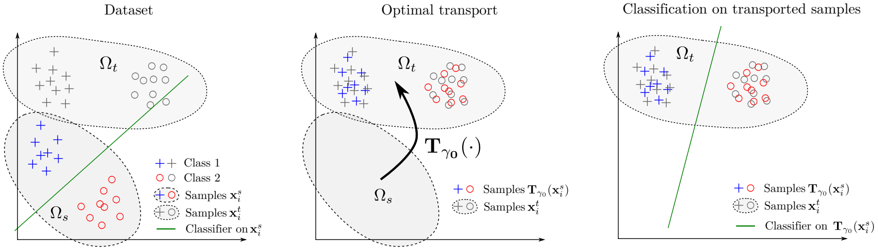
1. Estimate optimal transport between distributions.
2. Transport the training samples with barycentric mapping .
3. Learn a classifier on the transported training samples.



Step 1 : Estimate optimal transport between distributions.

- Choose the ground metric (squared euclidean in our experiments).
- Using regularization allows
  - Large scale and regular OT with entropic regularization [Cuturi, 2013].
  - Class labels in the transport with group lasso [Courty et al., 2016a].
- Efficient optimization based on Bregman projections [Benamou et al., 2015] and
  - Majoration minimization for non-convex group lasso.
  - Generalized Conditionnal gradient for general regularization (cvx. lasso, Laplacian).

OT for domain adaptation : Steps 2 & 3



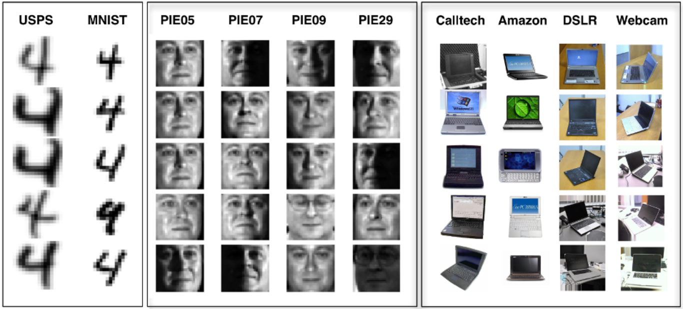
Step 2 : Transport the training samples onto the target distribution.

- The mass of each source sample is spread onto the target samples (line of γ_0).
- Transport using barycentric mapping [Ferradans et al., 2014].
- The mapping can be estimated for out of sample prediction [Perrot et al., 2016, Seguy et al., 2017].

Step 3 : Learn a classifier on the transported training samples

- Transported sample keep their labels.
- Classic ML problem when samples are well transported.

Visual adaptation datasets

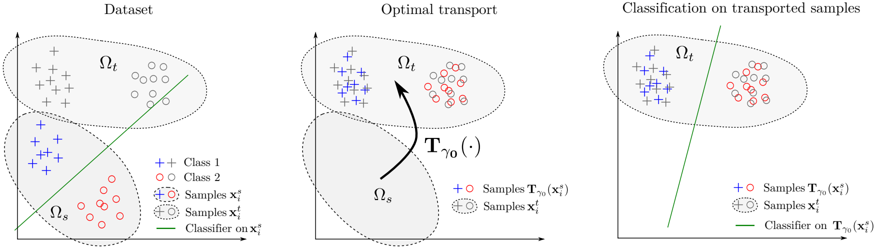


Datasets

- **Digit recognition**, MNIST VS USPS (10 classes, d=256, 2 dom.).
- **Face recognition**, PIE Dataset (68 classes, d=1024, 4 dom.).
- **Object recognition**, Caltech-Office dataset (10 classes, d=800/4096, 4 dom.).

Numerical experiments

- Comparison with state of the art on the 3 datasets.
- OT works very well on digits and object recognition.
- Works well on deep features adaptation and extension to semi-supervised DA.



Discussion

- Works very well in practice for large class of transformation [Courty et al., 2016a].
- Can use estimated mapping [Perrot et al., 2016, Seguy et al., 2017].

But

- Model transformation only in the feature space.
- Requires the same class proportion between domains [Tuia et al., 2015].
- We estimate a $T : \mathbb{R}^d \rightarrow \mathbb{R}^d$ mapping for training a classifier $f : \mathbb{R}^d \rightarrow \mathbb{R}$.

Joint distribution OT for domain adaptation (JDOT)

Joint distribution and classifier estimation

Objectives of JDOT

- Model the transformation of labels (allow change of proportion/value).
- Learn an optimal target predictor with no labels on target samples.
- Approach theoretically justified.

Joint distributions and dataset

- We work with the joint feature/label distributions.
- Let $\Omega \in \mathbb{R}^d$ be a compact input measurable space of dimension d and $\mathcal{C}$ the set of labels.
- Let $\mathcal{P}_s(X,Y) \in \mathcal{P}(\Omega \times \mathcal{C})$ and $\mathcal{P}_t(X,Y) \in \mathcal{P}(\Omega \times \mathcal{C})$ the source and target joint distribution.
- We have access to an empirical sampling $\hat{\mathcal{P}}_s = \frac{1}{N_s}\sum_{i=1}^{N_s} \delta_{\mathbf{x}_i^s,\mathbf{y}_i^s}$ of the source distribution defined by $\mathbf{X}_s = \{\mathbf{x}_i^s\}_{i=1}^{N_s}$ and label information $\mathbf{Y}_s = \{\mathbf{y}_i^s\}_{i=1}^{N_s}$.
- but the target domain is defined only by an empirical distribution in the feature space with samples $\mathbf{X}_t = \{\mathbf{x}_i^t\}_{i=1}^{N_t}$.

Joint distribution OT (JDOT)

Proxy joint distribution

- Let f be a $\Omega \rightarrow \mathcal{C}$ function from a given class of hypothesis $\mathcal{H}$.
- We define the following joint distribution that use f as a proxy of y

$$\mathcal{P}_t^f = (\mathbf{x}, f(\mathbf{x}))_{\mathbf{x} \sim \mu_t} \tag{5}$$

and its empirical counterpart $\hat{\mathcal{P}_t}^f = \frac{1}{N_t} \sum_{i=1}^{N_t} \delta_{\mathbf{x}_i^t, f(\mathbf{x}_i^t)}$.

Learning with JDOT
We propose to learn the predictor f that minimize :

$$\min_f \quad \left\{ W_1(\hat{\mathcal{P}}_s, \hat{\mathcal{P}_t}^f) = \inf_{\boldsymbol{\gamma} \in \Delta} \sum_{ij} \mathcal{D}(\mathbf{x}_i^s, \mathbf{y}_i^s; \mathbf{x}_j^t, f(\mathbf{x}_j^t)) \boldsymbol{\gamma}_{ij} \right\} \tag{6}$$

- Δ is the transport polytope.
- $\mathcal{D}(\mathbf{x}_i^s, \mathbf{y}_i^s; \mathbf{x}_j^t, f(\mathbf{x}_j^t)) = \alpha \|\mathbf{x}_i^s - \mathbf{x}_j^t\|^2 + \mathcal{L}(\mathbf{y}_i^s, f(\mathbf{x}_j^t))$ with $\alpha > 0$.
- We search for the predictor f that better align the joint distributions.

We provide a theoretical analysis of this choice. After introducing some notions:

Expected loss
The expected loss on a domain D and for a given predictor f is defined as

$$err_D(f) \stackrel{\text{def}}{=} \mathop{\mathbb{E}}_{(\mathbf{x},y)\sim\mathcal{P}_t} \mathcal{L}(y, f(\mathbf{x})).$$

Similarly we define a notion of agreement in D between two hypothesis functions f and g as $err_D(f,g) = \mathbb{E}_{(\mathbf{x})\sim D}\, \mathcal{L}(g(\mathbf{x}), f(\mathbf{x}))$.

We define a novel version of the Probabilistic Lipschitzness:

Probabilistic Lipschitzness [Urner et al., 2011, Ben-David et al., 2012]
Let $\phi : \mathbb{R} \to [0, 1]$. A labeling function $f : \Omega \to \mathbb{R}$ is ϕ-Lipschitz with respect to a distribution P over Ω if for all $\lambda > 0$

$$Pr_{x \sim P}\left[\exists y : [|f(x) - f(y)| > \lambda d(x, y)]\right] \leq \phi(\lambda).$$

Probabilistic Transfer Lipschitzness
Let μ_s and μ_t be respectively the source and target distributions. Let $\phi : \mathbb{R} \to [0, 1]$. A labeling function $f : \Omega \to \mathbb{R}$ and a joint distribution $\Pi(\mu_s, \mu_t)$ over μ_s and μ_t are ϕ-Lipschitz transferable if for all $\lambda > 0$:

$$Pr_{(\mathbf{x}_1, \mathbf{x}_2) \sim \Pi(\mu_s, \mu_t)}\left[|f(\mathbf{x}_1) - f(\mathbf{x}_2)| > \lambda d(\mathbf{x}_1, \mathbf{x}_2)\right] \leq \phi(\lambda).$$

Generalization bound (3)

Theorem 1

Let f be any labeling function of $\in \mathcal{H}$. Let $\Pi^* = \text{argmin}_{\Pi \in \Pi(\mathcal{P}_s, \mathcal{P}_t^f)} \int_{(\Omega \times \mathcal{C})^2} \alpha d(\mathbf{x}_s, \mathbf{x}_t) + \mathcal{L}(y_s, y_t) d\Pi(\mathbf{x}_s, y_s; \mathbf{x}_t, y_t)$ and $W_1(\hat{\mathcal{P}}_s, \hat{\mathcal{P}}_t^f)$ the associated 1-Wasserstein distance. Let $f^* \in \mathcal{H}$ be a Lipschitz labeling function that verifies the ϕ-probabilistic transfer Lipschitzness (PTL) assumption w.r.t. Π^* and that minimizes the joint error $err_S(f^*) + err_T(f^*)$ w.r.t all PTL functions compatible with Π^*. We assume the input instances are bounded s.t. $|f^*(\mathbf{x}_1) - f^*(\mathbf{x}_2)| \leq M$ for all $\mathbf{x}_1, \mathbf{x}_2$. Let $\mathcal{L}$ be any symmetric loss function, k-Lipschitz and satisfying the triangle inequality. Consider a sample of N_s labeled source instances drawn from $\mathcal{P}_s$ and N_t unlabeled instances drawn from μ_t, and then for all $\lambda > 0$, with $\alpha = k\lambda$, we have with probability at least $1 - \delta$ that:

$$\begin{aligned} \mathrm{err}_T(f) \ \leq \ & W_1(\hat{\mathcal{P}}_s, \hat{\mathcal{P}}_t^f) + \sqrt{\frac{2}{c'} \log(\frac{2}{\delta})} \left(\frac{1}{\sqrt{N_S}} + \frac{1}{\sqrt{N_T}} \right) \\ & + err_S(f^*) + err_T(f^*) + kM\phi(\lambda). \end{aligned}$$

- First term is JDOT objective function.
- Second term is an empirical sampling bound.
- Last terms are usual in DA [Mansour et al., 2009, Ben-David et al., 2010].

Optimization problem

$$\min_{f \in \mathcal{H}, \boldsymbol{\gamma} \in \Delta} \quad \sum_{i,j} \boldsymbol{\gamma}_{i,j} \left(\alpha d(\mathbf{x}_i^s, \mathbf{x}_j^t) + \mathcal{L}(y_i^s, f(\mathbf{x}_j^t)) \right) + \lambda \Omega(f) \tag{7}$$

Optimization procedure

- $\Omega(f)$ is a regularization for the predictor f
- We propose to use block coordinate descent (BCD)/Gauss Seidel.
- Provably converges to a stationary point of the problem.

γ update for a fixed f

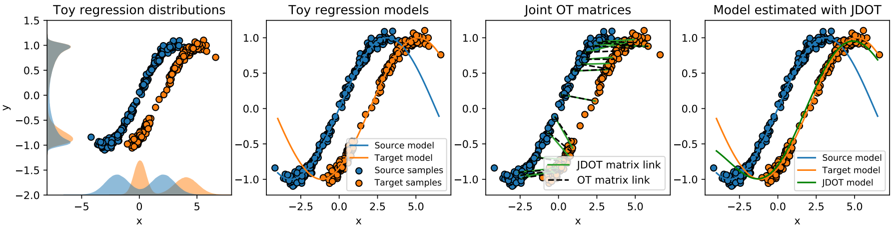
- Classical OT problem.
- Solved by network simplex.
- Regularized OT can be used (add a term to problem (7))

f update for a fixed γ

$$\min_{f \in \mathcal{H}} \quad \sum_{i,j} \boldsymbol{\gamma}_{i,j} \mathcal{L}(y_i^s, f(\mathbf{x}_j^t)) + \lambda \Omega(f) \tag{8}$$

- Weighted loss from all source labels.
- γ performs label propagation.

Regression with JDOT

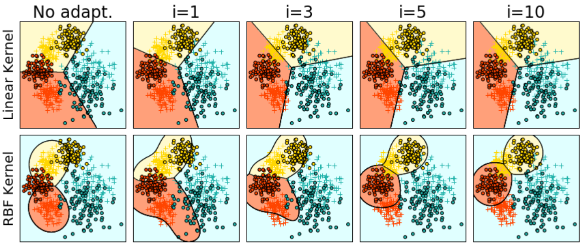


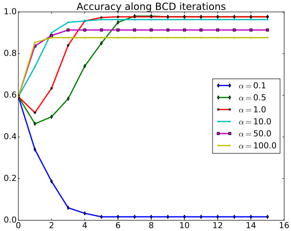
Least square regression with quadratic regularization
For a fixed γ the optimization problem is equivalent to

$$\min_{f\in\mathcal{H}} \quad \sum_j \frac{1}{n_t}\|\hat{y}_j - f(\mathbf{x}_j^t)\|^2 + \lambda\|f\|^2 \tag{9}$$

- $\hat{y}_j = n_t \sum_j \gamma_{i,j} y_i^s$ is a weighted average of the source target values.
- Note that this problem is linear instead of quadratic.
- Can use any solver (linear, kernel ridge, neural network).

Classification with JDOT





Multiclass classification with Hinge loss
For a fixed γ the optimization problem is equivalent to

$$\min_{f_k \in \mathcal{H}} \quad \sum_{j,k} \hat{P}_{j,k} \mathcal{L}(1, f_k(\mathbf{x}_j^t)) + (1 - \hat{P}_{j,k}) \mathcal{L}(-1, f_k(\mathbf{x}_j^t)) + \lambda \sum_k \|f_k\|^2 \qquad (10)$$

- $\hat{\mathbf{P}}$ is the class proportion matrix $\hat{\mathbf{P}} = \frac{1}{N_t} \boldsymbol{\gamma}^\top \mathbf{P}^s$.
- $\mathbf{P}^s$ and $\mathbf{Y}^s$ are defined from the source data with One-vs-All strategy as

$$Y_{i,k}^s = \begin{cases} 1 & \text{if } y_i^s = k \\ -1 & \text{else} \end{cases}, \qquad P_{i,k}^s = \begin{cases} 1 & \text{if } y_i^s = k \\ 0 & \text{else} \end{cases}$$

with $k \in 1, \cdots, K$ and K being the number of classes.

Caltech-Office classification dataset

Domains	Base	SurK	SA	OT-IT	OT-MM	JDOT
caltech→amazon	92.07	91.65	90.50	89.98	**92.59**	91.54
caltech→webcam	76.27	77.97	81.02	80.34	78.98	**88.81**
caltech→dslr	84.08	82.80	85.99	78.34	76.43	**89.81**
amazon→caltech	84.77	84.95	85.13	85.93	**87.36**	85.22
amazon→webcam	79.32	81.36	**85.42**	74.24	85.08	84.75
amazon→dslr	86.62	87.26	**89.17**	77.71	79.62	87.90
webcam→caltech	71.77	71.86	75.78	**84.06**	82.99	82.64
webcam→amazon	79.44	78.18	81.42	89.56	90.50	**90.71**
webcam→dslr	96.18	95.54	94.90	**99.36**	**99.36**	98.09
dslr→caltech	77.03	76.94	81.75	**85.57**	83.35	84.33
dslr→amazon	83.19	82.15	83.19	**90.50**	**90.50**	88.10
dslr→webcam	96.27	92.88	88.47	**96.61**	**96.61**	**96.61**
Mean	83.92	83.63	85.23	86.02	86.95	**89.04**
Avg. rank	4.50	4.75	3.58	3.00	2.42	**2.25**

- Classical dataset [Saenko et al., 2010] dedicated to visual adaptation.
- Feature extraction by convolutional neural network [Donahue et al., 2014].
- Comparison with Surrogate Kernel [Zhang et al., 2013], Subspace Alignment [Fernando et al., 2013] and OT Domain Adaptation [Courty et al., 2016b].
- Parameter selected via reverse cross-validation [Zhong et al., 2010].
- SVM (Hinge loss) classifiers with linear kernel.
- Best ranking method and 2% accuracy gain in average.

Amazon Review Classification dataset

Domains	**NN**	**DANN**	**JDOT** (mse)	**JDOT** (Hinge)
books→dvd	0.805	**0.806**	0.794	0.795
books→kitchen	0.768	0.767	0.791	**0.794**
books→electronics	0.746	0.747	0.778	**0.781**
dvd→books	0.725	0.747	0.761	**0.763**
dvd→kitchen	0.760	0.765	0.811	**0.821**
dvd→electronics	0.732	0.738	0.778	**0.788**
kitchen→books	0.704	0.718	**0.732**	0.728
kitchen→dvd	0.723	0.730	0.764	**0.765**
kitchen→electronics	**0.847**	0.846	0.844	0.845
electronics→books	0.713	0.718	0.740	**0.749**
electronics→dvd	0.726	0.726	**0.738**	0.737
electronics→kitchen	0.855	0.850	0.868	**0.872**
Mean	0.759	0.763	0.783	**0.787**

- Dataset aim at predicting reviews across domains [Blitzer et al., 2006].
- Comparison with Domain adversarial neural network [Ganin et al., 2016a].
- Classifier f is a neural network with same architecture as DANN.
- JDOT has better accuracy, classification loss is better than mean square error.

Wifi localization regression dataset

Domains	KRR	SurK	DIP	DIP-CC	GeTarS	CTC	CTC-TIP	JDOT
t1 $\rightarrow$ t2	80.84±1.14	90.36±1.22	87.98±2.33	91.30±3.24	86.76 ± 1.91	89.36±1.78	89.22±1.66	**93.03 ± 1.24**
t1 $\rightarrow$ t3	76.44±2.66	**94.97±1.29**	84.20±4.29	84.32±4.57	90.62±2.25	94.80±0.87	92.60 ± 4.50	90.06 ± 2.01
t2 $\rightarrow$ t3	67.12±1.28	85.83 ± 1.31	80.58 ± 2.10	81.22 ± 4.31	82.68 ± 3.71	87.92 ± 1.87	**89.52 ± 1.14**	86.76 ± 1.72
hallway1	60.02 ±2.60	76.36 ± 2.44	77.48 ± 2.68	76.24± 5.14	84.38 ± 1.98	86.98 ± 2.02	86.78 ± 2.31	**98.83±0.58**
hallway2	49.38 ± 2.30	64.69 ±0.77	78.54 ± 1.66	77.8± 2.70	77.38 ± 2.09	87.74 ± 1.89	87.94 ± 2.07	**98.45±0.67**
hallway3	48.42 ±1.32	65.73 ± 1.57	75.10± 3.39	73.40± 4.06	80.64 ± 1.76	82.02± 2.34	81.72 ± 2.25	**99.27±0.41**

- Objective is to predict position of a device on a discretized grid [Zhang et al., 2013].
- Same experimental protocol as [Zhang et al., 2013, Gong et al., 2016].
- Comparison with domain-invariant projection and its cluster regularized version ([Baktashmotlagh et al., 2013], **DIP** and **DIP-CC**), generalized target shift ([Zhang et al., 2015], **GeTarS**), and conditional transferable components, with its target information preservation regularization ([Gong et al., 2016], **CTC** and **CTC-TIP**).
- JDOT solves the adaptation problem for transfer across device (10% accuracy gain on Hallway).

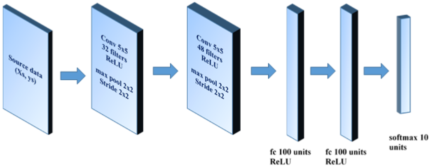
Large scale JDOT

- JDOT do not scale well to large datasets/ deep learning.
- Use minibach for computing the transport in the primal [Genevay et al., 2017].
- Evaluate batch-local couplings on (sufficiently large) couples of random (without replacement) batches in source and target domain
- update f from these couplings

Algorithm : Deep JDOT

input Source data X^s, y^s, Targte data X^t
  for BCD Iterations **do**
    for each Source/Target minibatch **do**
      Solve OT with JDOT loss
      Perform label propagation on minibatch
    end for
    Update model f on one epoch
  end for

Large scale datasets



Description	MNIST→ USPS	USPS→MNIST	SVHN→MNIST	MNIST→ MNIST-M
Source samples	60000	9298	73257	60000
Target samples	9298	60000	60000	60000
height/width	16×16	16×16	32×32×3	28×28×3

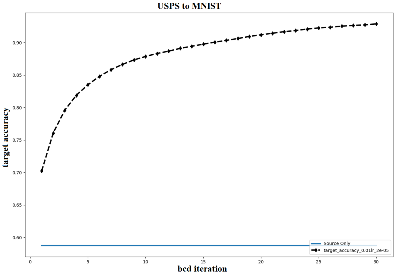
- Four cross domain digits datasets: MNIST, USPS, SVHN, MNIST-M .

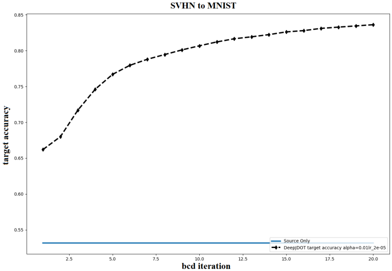
Experimental Results for large scale JDOT

Methods	MNIST→ USPS	USPS→MNIST	SVHN→MNIST	MNIST→ MNIST-M
Source only (SO)	86.18	58.73	53.15	59.52
DeepCoral [Sun and Saenko, 2016]	88.43 (22.0)	85.02 (64.6)	69.61 (35.6)	62.18 (0.07)
MMD [Long and Wang, 2015]	89.89 (36.3)	79.19 (50.3)	53.27 (0.01)	52.53 (-19.1)
DANN [Ganin et al., 2016b]	89.06 (28.2)	87.03 (70.0)	73.85* (44.7)	76.63 (46.6)
ADDA [Tzeng et al., 2017]	91.22 (49.3)	79.98 (52.2)	76.0* (49.4)	**79.16 (53.5)**
DeepJDOT	**91.50 (52.01)**	**91.21 (79.82)**	**83.62 (65.85)**	67.84 (22.67)
Train on Target (TO)	96.41	99.42	99.42	96.21

- Accuracy in % of the DA methods.
- The values in () represent the coverage gap between SO (source only) and TO (golden performance if the model is learnt on target labelled data), $\frac{DA-SO}{TO-SO}$.
- DeepJDOT is better in 3 out of 4 DA problems.
- Plots represent test performances along the BCD iterations.

Experimental Results for large scale JDOT





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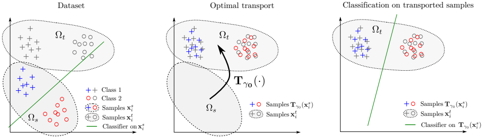
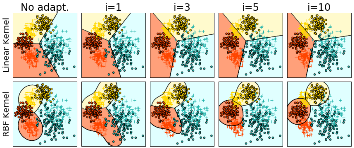
Conclusion





Optimal transport for DA

- Model transformation of the features.
- Conditional distribution preserved.
- Mapping between distributions.
- Learn classifier on the transported samples.

Joint distribution OT for DA

- Model transformation of the joint distribution.
- General framework for DA.
- Theoretical justification with generalization bound.

Next ?

- SGD OT on the semi-dual [Genevay et al., 2016] or dual [Seguy et al., 2017].
- Learn simultaneously the best feature representation [Shen et al., 2017].

Thank you

Python code available on GitHub:

`https://github.com/rflamary/POT`

- OT LP solver, Sinkhorn (stabilized, $\epsilon-$scaling, GPU)
- Domain adaptation with OT.
- Barycenters, Wasserstein unmixing.
- Wasserstein Discriminant Analysis.

Python code for JDOT on GitHub:

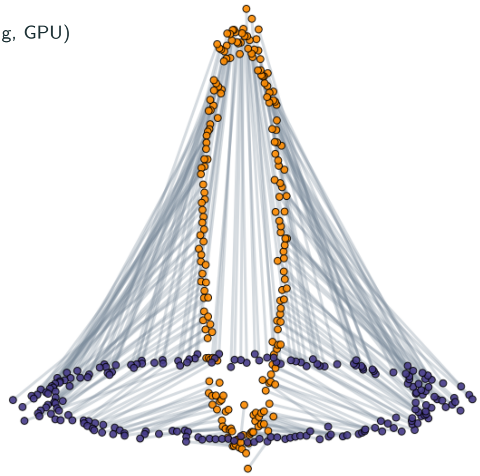
`https://github.com/rflamary/JDOT`

Papers available on my website:

`https://remi.flamary.com/`

Post docs available in:

Nice, Rouen, Rennes (France)



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